Home work week 1

In the lecture we discussed reversible work by one mole of ideal gas. Atoms in an ideal gas only have kinetic energy and hence the internal energy of the ideal gas only depends on temperature T. Thus, $U(T)$ is a state function and any (Infinitesimally small (d)) changes in the internal energy (and thus temperature) are due to heat exchange and/or work:

$$dU = dQ + dW \tag{1}$$

Heat Q and work W are *not* state functions (*i.e.* their values depend on the path, whereas the value of state function $U(T)$ does not). For an ideal gas the reversible work is

$$dW = -pdV \tag{2}$$

and the equation of state is

$$pV = RT \tag{3}$$

1. Derive an analytic expression for the *reversible* work when the ideal gas is expanded (or compressed) from V_1 to V_2 at a constant temperature T (isothermal expansion or compression) and answer the following questions:

   (a) How does the work depend on the temperature?

   (b) What is the net work if the ideal gas is first isothermally expanded and then isothermally compressed at same temperature?

   (c) What is the net work if the ideal gas is first isothermally expanded from V_1 to V_2 at a higher temperature (T_{hot}) and then isothermally compressed back at a lower temperature (T_{cold}) ?

2. Derive an analytic expression for the *reversible* work when the ideal gas is

expanded or compressed while isolated from the rest of the world. During the so-called *adiabatic* expansion or compression, no heat is exchanged with the environment: $dQ = 0$ and hence the temperature changes from T_1 to T_2.

3. Show that for one mole of ideal gas (equation of state: $PV = RT$) in isolation (*i.e.*, adiabatic $dQ = 0$) we have that PV^{γ} = constant during reversible expansion, where $\gamma = C_P/C_V$. Use that with $dQ = 0$ the change in the energy (U) of the ideal gas (which is not constant if V or P are varied; why?) is $dU = dW = -PdV$ and that U depends on T only. Further hints are that the heat capacity of the ideal gas at constant volume is $C_V = \left(\frac{dQ}{dT}\right)_V = \left(\frac{dU}{dT}\right)_V$ and heat capacity at constant pressure is $C_P = \left(\frac{(dQ}{dT}\right)_P = \left(\frac{dU}{dT}\right)_P + R$.

Hint: $\int \frac{1}{x} dx = \ln x$